GAS DYNAMICAL MODEL OF LINEAR LIGHTNING DISCHARGE

A. A. ANIKYEV[1*] AND V. I. PUSTOVOIT[2]

[1] Bauman Moscow State Technical University, 5/1, 2-d Baumanskaya St., 105005, Moscow, Russia
[2] Scientific and Technological Center of Unique Instrumentation RAS, 15 Butlerova St., 117342 Moscow, Russia
*aaanikyev@mail.ru

ABSTRACT

The hydrodynamic model of movement of the charged particles in a stream, expanding applicability of the initial model was studied. In present paper the analysis of various approximations 1D of model is carried out and phase trajectories of the electric field currency induced by a stream and density of particles in a stream at various speeds of excitations are constructed.

1. INTRODUCTION

Atmospheric discharges represent surprisingly beautiful phenomena which observed as in ground layers at the level of thunderclouds in the direction to the earth, and ascending from tops of clouds through the troposphere to an ionosphere – sprites, blue and Giant jets, elves. Scales of discharges cover areas from 12-20 km up to 40-90 km [1-3]. The main reason of all these phenomena is the process of division of a charge, increase of electric field and the subsequent breakdown of ionized media. In itself this phenomenon carries random character in time and space, but preparation of division of a charge and the subsequent increase of a field is process natural which reason are set of various physical factors. As the main processes leading to emergence of a linear lightning, two mechanisms, in detail stated, for example, in fine work [2] are offered. In one of them conventional breakdown on the electrons warmed in electric field is offered. The second mechanism – breakdown on run-away electrons – is energetically more favorable as assumes much smaller values of a field for acceleration of electrons and creation of avalanches. In this work new approach to an explanation of the physical mechanism of emergence of the discharge, based on hydrodynamic consideration of streams of the charged particles in the thunderstorm cloud caused by the natural reasons of convection and capture of charged particles by an ascending streams in terrestrial electric field is offered. Existence of hydrodynamic streams of the charged particles in slightly ionized gas can lead to development of instability in relation to non-uniform distribution of charges in a cloud, to increase of electric field and emergence of local areas with the electric fields exceeding value of breakdown at this altitude.

2 . THEORY

We assume that we deal with the slightly ionized dense gas consisting of electrons, ions and not charged particles. Proceeding from estimates of concentration of the charged particles about 10^4 cm^{-3} and assuming approximately a constant temperature in a layer of the ground atmosphere at altitude of thunderclouds, it is possible to estimate the Debye radius $r_D = v_T/\omega_n \approx 4$ cm that exceeds distance between the charged particles and therefore at smaller distances there is no Coulomb screening of the charged particles. Here $v^2_T = kT/m$ – the thermal velocity of electrons, and $\omega_n = 10^{12}$ s^{-1}– the characteristic frequency of fluctuations of the charged particles. As the main equations describing dynamics of streams of charged particles, we use Euler's equation, the equation of continuity and Poisson's equation for model slightly ionized gas. In a storm cloud in the terrestrial electric field we will accept initial distribution of the charged areas negative in the lower part and positive in the top part though observed distribution of a charge is much more complicated [2,4]. Temporary and spatial distribution of electric field in an one-dimensional case, will be described then by system of the equations:

$$-enE-\frac{\partial P}{\partial x}-mn\nu(\upsilon-\upsilon_0)=mn\frac{\partial\upsilon}{\partial t}+mnv\frac{\partial\upsilon}{\partial x},$$
$$\frac{\partial n}{\partial t}+\frac{\partial(n\upsilon)}{\partial x}=0,$$
$$\varepsilon_0\frac{\partial E}{\partial x}=4\pi e(N_0-n),$$
$$J=en\upsilon \tag{1}$$

Here the following designations are introduced: e – charge of a particle, m – its mass, υ – the speed of a particle, ν - the frequency of impacts of a particle with all scattering centers, n – concentration of particles, N_0 – concentration of particles of an opposite sign, υ_0 – the speed of neutral particles, ε_0 – dielectric permeability of media, P – gas pressure, in adiabatic approach. We will consider in the beginning isothermal process at scales into a thundercloud and on times previous to formation of the leader. Then pressure can be presented in the form of P = kTn. In this approach we don't consider mechanical viscosity, but electro dynamical viscosity is considered partially, introduction of the member of collisions $mn\nu(\upsilon-\upsilon_0)$. The mechanism of a recharge of neutral particles and ions can be considered by addition in the right member of equation of a continuity of integral of collisions. As a first approximation we won't consider recharge process. We will write down system (1) in a dimensionless form, using the following designations: $\xi = x/l$, $\tau = t/\tau_0$, $y = eEl/(kT)$, $\rho = nl^3$, $\rho_0 = N_0l^3$, $u = \upsilon/(\nu l)$, $u_0 = \upsilon_0/(\nu l)$, were $l=e\wedge 2/kT$ - characteristic distance at which energy of Coulomb interaction is equal to thermal energy of particles;

$\tau_0=\frac{m\nu l^2}{kT}=\frac{\upsilon_D{}^2}{\upsilon_T{}^2\nu}\equiv\frac{l^2}{D}$ - - time, during which non equilibrium concentration of particles is leveled due to diffusion (υ_D – the speed of diffusion, υ_T – the thermal speed of particles, D=kT / (mν) – diffusion coefficient). The received system will write in a form:

$$-\rho y-\frac{\partial\rho}{\partial\xi}-\tau_0\nu\rho(u-u_0)=\rho\frac{\partial u}{\partial\tau}+\tau_0\nu\rho u\frac{\partial u}{\partial\xi}$$
$$\frac{\partial\rho}{\partial\tau}+\tau_0\nu\frac{\partial(\rho u)}{\partial\xi}=0 \tag{2}$$
$$\frac{\partial y}{\partial\xi}=-\frac{4\pi}{\varepsilon_0}(\rho-\rho_0)$$
$$j=\rho u$$

For the dimensionless density of a stream of particles $j=Jl^2/(e\nu)$ designation is introduced. Having used the continuity equation, we will exclude speed from this system of the equations. After simple transformations we will receive system of the equations concerning two variables – density of particles and electric field in a gas stream:

$$\begin{cases}(y_0-y)\cdot\rho-\frac{\partial\rho}{\partial\xi}-c_1(\tau)-\frac{\varepsilon_0}{4\pi}\cdot\frac{\partial y}{\partial\tau}=\frac{1}{\tau_0\nu}\cdot\frac{\partial}{\partial\tau}[\\ c_1(\tau)+\frac{\varepsilon_0}{4\pi}\cdot\frac{\partial y}{\partial\tau}]+\frac{1}{\tau_0\nu}\cdot\frac{\partial}{\partial\xi}\left[\frac{1}{\rho}\cdot\left(c_1(\tau)+\frac{\varepsilon_0}{4\pi}\cdot\frac{\partial y}{\partial\tau}\right)^2\right]\\ \frac{\partial y}{\partial\xi}=-\frac{4\pi}{\varepsilon_0}\cdot(\rho-\rho_0)\end{cases} \tag{3}$$

Here $y_0 = \tau_0\nu u_0$ designation is introduced. The integration constant $c_1(\tau)$, received by us after integration of the equation of a continuity on coordinate is defined by values of density, speed of a stream and change of electric field on thundercloud border a ratio:

$$c_1(\tau)=\tau_0\nu\rho(\xi_0,\tau)u(\xi_0,\tau)-\frac{\varepsilon_0}{4\pi}\cdot\frac{\partial y(\xi_0,\tau)}{\partial\tau} \tag{4}$$

This value is the continuous function of time addressing in zero on border, and differ from zero out of it in view of the fact that change of electric field in this point of space during a period is compensated by density of a stream of the charged particles through thundercloud border. Tangential components of a rotor of a vector of magnetic field on border don't undergo jump, and have not influence on one-dimensional movement of current. We will introduce the new ζ variable connected with old variables by coordinate of ξ and time of τ by relation of ζ = ξ - Vτ. Considering ratios

$$\frac{\partial}{\partial\xi}=\frac{d}{d\zeta},\quad \frac{\partial}{\partial\tau}=-V\frac{d}{d\zeta}$$

we will write down system (4) finally in form:

$$\begin{cases} \frac{d\rho(\zeta)}{d\zeta}\cdot\left[\rho^2(\zeta)-\frac{(c_1(\zeta)-V\rho_0)^2}{\tau_0\nu}\right]+\rho^2(\zeta)\cdot \\ \left[c_1(\zeta)+V[\rho(\zeta)-\rho_0]+\rho(\zeta)[y(\zeta)-y_0]-\frac{V}{\tau_0\nu}\cdot\frac{dc_1(\zeta)}{d\zeta}\right]=0 \\ \frac{dy(\zeta)}{d\zeta}=-\frac{4\pi}{\varepsilon_0}\cdot[\rho(\zeta)-\rho_0] \end{cases}$$

(5)

The system (5) describes distribution of density of a stream and electric field in one-dimensional approach and at the set function $c_1(\zeta)$, determined by conditions on border. She allows to calculate distribution of electric field and density of a stream of gas at parameter V change – speeds of excitation of a field concerning the system of coordinates moving with a speed of a stream of gas. In $\rho_0 = 0$ approximation system it is possible to integrate and receive the equation

$$\frac{dz}{d\zeta}=-\frac{1}{4}(z^2-2b\zeta-2c-a^2)$$
$$\pm\frac{1}{4}\sqrt{(z^2-2b\zeta-2c-a^2)^2-16d^2}$$

$$z=y+a,\ a=V-y_0,\ b=\frac{4\pi}{\varepsilon_0}c_1,\ c=b\cdot\frac{V}{\tau_0\nu},\ d^2=\frac{1}{\tau_0\nu}\cdot b^2$$

Considering that d2 ≤ z2 we will receive Rikkati's equation

$$\frac{dz}{d\zeta}=-\frac{1}{2}(z^2-2b\zeta-2c-a^2) \qquad (6)$$

which has the decision

$$z(t)=-\frac{1}{A}\cdot\frac{u_t'}{u},\ \text{где}$$

$$u(t)=\sqrt{t}\cdot\left[C_2\cdot J_{\frac{1}{2q}}\left(\frac{1}{q}\sqrt{AB}t^q\right)+C_3Y_{\frac{1}{2q}}\left(\frac{1}{q}\sqrt{AB}t^q\right)\right]$$

(7)

$J_m(x),\ Y_m(x)$ - Bessel functions of the first and second kind. $q=\frac{n+2}{2}$, in our case n = 1. The figure of the dependence of an electric field as functions of coordinate is submitted in figure 1.

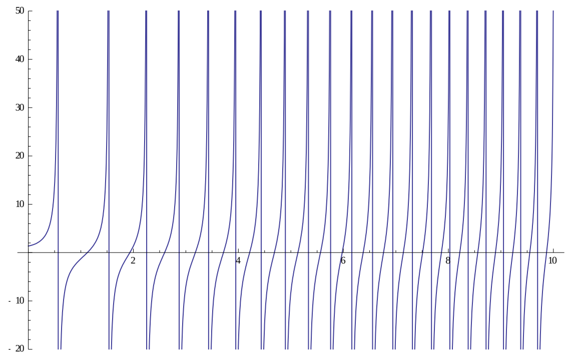

Fig. 1.The dependence of z (t) at values of the constants C1 = 1.2, C2 = 2.3, found from boundary conditions. The constant b = 10, a variable t runs values from 0 to 10.

In the received approach we have essential increase of amplitude of electric field at stream movement, and breakdown of an electric field with the poles which are condensing in process of increase of coordinate of a stream are observed. In our further statement we will consider system (5) without any approximations. As the first step we express dimensionless concentration of ρ(ζ) from the second equation and substitute it in the first equation. Then we receive the equation

$y_{\zeta\zeta}(A_1y_\zeta^2 + B_1y_\zeta + C_1) = (A_2 + y)\, y_\zeta^3 + (B_2 + y)y_\zeta^2 + (C_2 + y)y_\zeta + D$ (8)

If we introduce the new variable w (y)=y_ζ, including y as an independent variable then the equation (8) can be write in a form:

$w_yw(A_1w^2 + B_1w + C_1) = (A_2 + y)\, w^3 + (B_2 + y)w^2 + (C_2 + y)w + D$

If to rewrite it in a form

$dy[(A_2 + y)\, w^3 + (B_2 + y)w^2 + (C_2 + y)w + D] = dw\, [w(A_1w^2 + B_1w + C_1)],$ (9)

that it can be integrated after finding of an integrating multiplier. The solution of this equation takes a lot of place, therefore we will provide only the schedules received by us at the solution of this equation in y (ζ). In figures 2 - 5 dependences of the given concentration, amplitude of electric field versus coordinate, entry conditions are presented and at various values of excitation velocity in a stream of ionized gas.

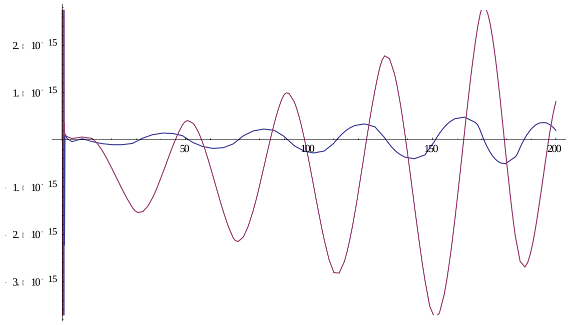

Fig. 2. Evolution of electric field in a stream (the red line) and the derivative of the field of a stream (the blue line) at value of speed of excitation in a stream of V = 151.021, constructed in the coordinate range of ξ = [0 – 200].

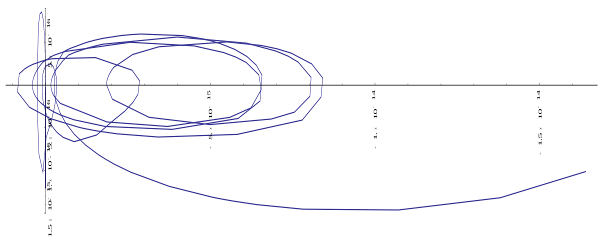

Fig. 3. Phase trajectory of electric field amplitude y corresponding to figure 2 data y' (y) at values of the specified coordinate of ξ = [0 – 200].

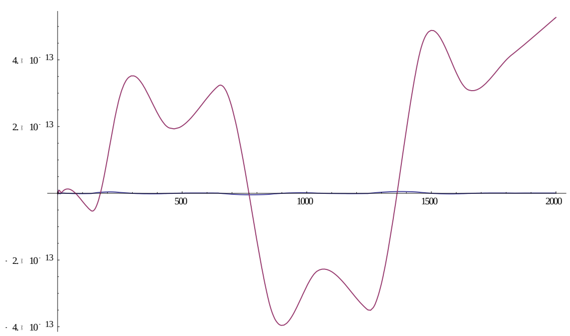

Fig. 4. Evolution of density of a stream (the red line) and the derivative density of a stream (the blue line) at value of speed of excitation in a stream of V = 4515.54. Areas of change of the ξ variable = [0 – 2000].

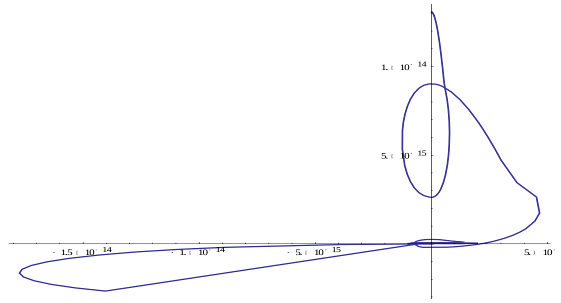

Fig. 5. Phase trajectory of density of a stream of ρ'(ρ) corresponding to figure 4 data at values of the specified coordinate of ξ = [0 – 1000].

Amplitude of electric field increases on 5 – 7 orders, remaining limited as the speed of stream increases. As the results of our study it is possible to make the conclusion that the used gas dynamic model of formation of the linear atmospheric discharge can even describe the emergence of the leader connected with transformation of kinetic energy of a stream in energy of the discharge at the expense of creation of high electric fields in a stream in the considered one-dimensional case.

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