3D TESTS OF POISSON EQUATION SOLVERS FOR STREAMER SIMULATION

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ABSTRACT

The main goal of the present communication is to test, in terms of computing time cost and precision, Poisson equation solvers in a cubic 3D configuration for further applications in 3D streamer simulation using High Performance Parallel Computing. The Poisson equation is discretized with the Finite Volume Method and the cubic domain is divided into $n \times n \times n$ nodal points (n being 50, 100 or 200). The chosen configuration could be a basic block of a larger discretized domain distributed on several processors. The upper and the lower planes of the cubic domain are respectively the anode and the cathode, while the other lateral surfaces are open space. The calculation is either performed for the geometric field (Laplace equation) or takes into account the propagation of an analytical space charge density.

1. INTRODUCTION

For a short high pulsed voltage condition, the breakdown corona discharge regime is characterized by the propagation of many micro-discharges, presenting a large branching structure [1]. However, for a DC high voltage condition, the branching structure rapidly vanishes and a mono-filament micro-discharge occurs, characterized by the development of a primary and a secondary streamer [2]. In the field of the fluid model approximation, this kind of DC micro-discharge can be simulated using a 2D(r,z) cylindrical grid domain that reproduces the symmetry of the mono-filament (see e.g. [3] and [4]). Obviously, for high pulse voltage conditions, the cylindrical symmetry is completely broken due to the complex branching structure of the global discharge. It is therefore necessary to upgrade the simulations to a Cartesian 3D geometry. Nevertheless, it remains a great challenge to simulate in 3D a complete corona breakdown because of the multi-scale nature of the micro-discharges [5]. Indeed, a strong spatial grid refinement is necessary in order to capture the dynamics of the multiple streamer heads and the interaction between each micro-discharge. In the last decades, increasing computer power has permitted the 3D simulation of micro-discharges using classical first order fluid models and/or hybrid calculations [6-9]. These previous works were devoted to the simulation of the very beginning of the discharge development, the 3D grid refinement optimization, and the effects of several physical models and assumptions on the propagation and the splitting of an initial streamer channel. The new challenge is to simulate the development and propagation of a complete breakdown corona discharge, involving some tens of micro-discharges interacting with each other during hundreds of nanoseconds over a spatial scale on the order of centimetres. This new challenge can be overcome thanks to the use of High Performance Parallel Computing. However, significant attention must be paid to the performance of the numerical solvers used and their ability to maintain their efficiency as the number of individual compute nodes is

increased. Indeed, in the case of distributed memory parallelisation using the Message Passing Interface (MPI) library, the decrease in computing time with an increase in the number of compute nodes should be maintained as long as possible.

In the present work and as a first step, we have studied the performance of several elliptic equation solvers on one compute node of the High Power Computer Hyperion [10]. We focus our study on the solution of elliptic equations because they are essential for the calculation of the potential and in some cases for the photo-ionisation processes. Furthermore, their numerical processing represents more than 90% of the total computing time needed to solve the set of equations involving in the classical first order fluid model.

The first section of the present paper describes the simulation conditions while the second and the third sections are respectively devoted to comparing the results obtained and to presenting the main conclusions and perspectives of this work.

2. SIMULATION CONDITIONS

We have tested several elliptic equation solvers for the calculation of the electric potential using the Poisson equation with or without (Laplace equation) the addition of the propagation of an analytical space charge density. The Poisson equation is discretized with the Finite Volume Method on a Cartesian cubic domain of $1mm^3$ divided into n^3 nodal points, n being equal to 50, 100 or 200. The chosen configuration could be a basic block of a larger discretized domain distributed on several compute nodes. The upper and the lower planes of the cubic domain are respectively the anode (6kV) and the cathode (0kV) electrode, while the other lateral surfaces are open space (i.e. Neumann conditions are applied with the potential derivative equal to zero on the direction perpendicular to each lateral surface). The space charge density has a Gaussian shape with a full-width at half-maximum equal to 50µm. The maximum propagates along the vertical symmetry axis of the cube from 0.4mm until 0.8mm with a velocity of $10^6 m.s^{-1}$.

After the discretization on the 3D Cartesian domain, the elliptic Poisson equation can be expressed in the form of a linear matrix system:

$$Av = b \tag{1}$$

where A is a sparse matrix (of $n^3 \times n^3$ dimension), v the potential vector and b the space charge density vector involving the potential boundary conditions.
In order to solve this linear matrix system, we use the iterative solvers proposed in the LIS library [11]. We focus our study on the iterative methods because they can be very efficient, especially for time dependant problems where the solution at a previous time, t, can be used as an initial solution to begin the new iteration process and converge rapidly towards the solution at time $t + \Delta t$. After preliminary tests made with the twenty-two proposed methods in the LIS library, we compare the efficiency of 3 selected ones: IDR(s), BiCSTAB(l) and the classical SOR(ω). In all cases, the convergence criterion is defined as follow:

$$\frac{\|Av - b\|}{\|b\|} < \varepsilon \tag{2}$$

where $\| \ \|$ stands for the norm 2 of the corresponding vectors. ε is the chosen precision, varying from 10^{-8} to 10^{-6}. Furthermore, the maximum number of iterations was fixed at 9000. It means that the solver failed when the solution has not converged towards the chosen precision after a maximum of 9000 iterations.

3. RESULTS AND ANALYSIS

The three selected methods depend on specific parameters s, l or ω. Therefore, preliminary tests were made in order to choose the best values that accelerate the numerical convergence of each solver. It was found that taking s=3, l=2 and ω=1.6 enhances the efficiency of the methods in terms of precision and computing time.

3.1 Solution of the Laplace equation

Fig. 1, 2 and 3 show the efficiency of the 3 solvers in terms of computing time versus precision for respectively n=50, 100, and 200. The method SOR(1.6) failed for n=100 and $\varepsilon = 10^{-8}$ and for all precisions when n=200. The

results indicate that the general trends of IDR(3) and BiCSTAB(2) are very similar, with BiCSTAB(2) converging slightly more rapidly for a given precision (excepted for n=50 and $\varepsilon = 10^{-8}$).

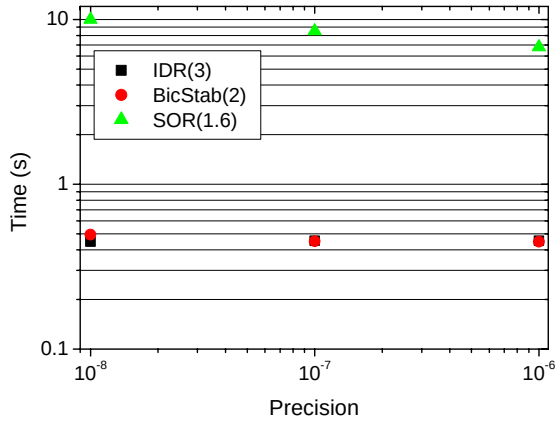

Fig.1: Efficiency of the 3 solvers for n=50

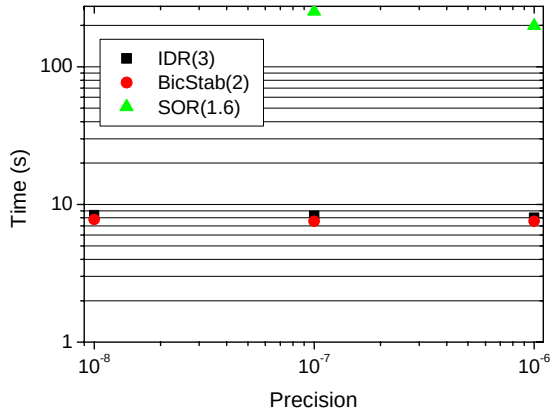

Fig.2: Efficiency of the 3 solvers for n=100

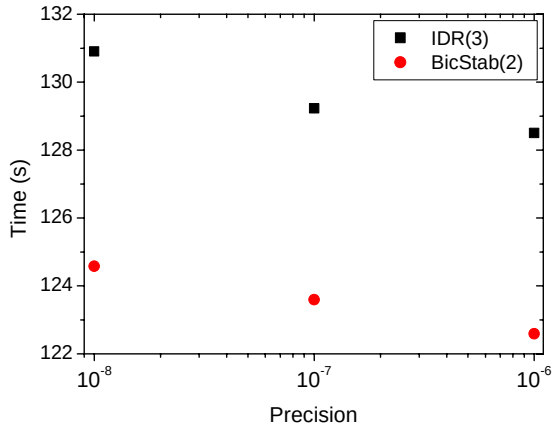

Fig.3: Efficiency of 2 solvers for n=200

It is interesting to note that a gain of 100 in the precision (i.e. by reducing the precision from 10^{-6} to 10^{-8}) produces a very small increase of computing time especially for the IDR(3) and the BiCSTAB(2) methods. For example, with n=200 in Fig.3, the IDR(3) and BiCSTAB(2) solvers require only 1.6% and 1.9% more computing time to converge from 10^{-6} to 10^{-8}. In both cases, this corresponds to fewer than 10 additional iterations. Still considering the two more efficient methods, and with $\varepsilon = 10^{-8}$, it can be seen that multiplying the number of cells by 8 increases the computing time by a factor of about 16. Indeed, from 50^3 to 100^3 ($=8\times50^3$) to 200^3($=8\times100^3$), the mean computing time for both methods increases from 0.49s to 8.07s($\approx16.5\times0.49$s) and from 8.07s to 127s($\approx15.7\times8.07$s), respectively.

3.2 Solution of the Poisson equation

In this section, and following the previous results, only the case n=50 is chosen for a comparison of the 3 tested methods. Prior to considering the propagation of the space charge density profile, we have tested the convergence of the 3 solvers when the maximum of the Gaussian space charge is fixed on the vertical cube axis, 0.4mm above the cathode plane. The results obtained are presented in Fig.4. For example, Fig.4 shows that for a precision of $\varepsilon = 10^{-7}$, the IDR(3) solver requires nearly twice as many computing time to converge to a solution for the Poisson equation as for the Laplace equation.

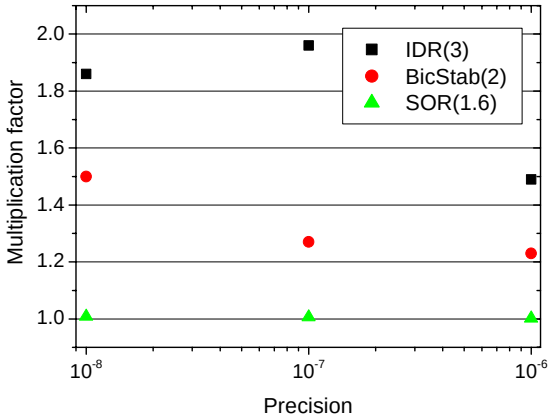

Fig.4: Relationship between the computing time required for solving the Poisson equation versus the computing time needed for solving Laplace equation (n=50)

By comparison, the SOR(1.6) method remains very stable and, for a given precision, requires the a similar computing time for both equations. It is noteworthy that Fig.4 is also representative of the mean computing time needed for the three solvers to converge at each time step t when the Gaussian space charge density propagates from

0.4mm to 0.8mm along the vertical tube axis. It means that for n=50, the initial solution used to start the iteration process at a given time step t has no influence on the computing time. Indeed, the computing time should be reduced, if the initial solution chosen at time $t - \Delta t$ is not so far from the solution at time t so that the number of iterations is significantly reduced. Further tests are required for n=100 and n=200 in order to see if the increase of the space grid resolution will have an influence on the mean computing time at each time step evolution.

4. CONCLUSION AND PERSPECTIVES

The present preliminary results indicated that the BiCSTAB(2) iterative method presents the best efficiency in terms of precision versus computing time whatever the tested configuration (Laplace or Poisson equation with or without the propagation of a space charge density). However, more recent simulations have shown that the SOR(ω) method is very sensitive to the value of ω which also depends on the value of n (i.e. the space grid definition). The further works will be focussed on the SOR(ω) optimisation, the adjunction of a direct method (e.g. MUMPS) and the study of the computation time efficiency with the number of computation nodes.

ACKNOWLEDGMENT

This work was performed using HPC resources from CALMIP (Grant 2011-[P1053]) [10]

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[11] http://www.ssisc.org/lis/index.en.html