BALL LIGHTNING: AN ASSESSMENT OF THE KNOWN PHYSICAL FEATURES AND A SUGGESTED MATHEMATICAL MODEL

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ABSTRACT

A brief review of the salient features of ball lightning is given, listing what appear to be the essential physical effects of these elusive and varied happenings. This is followed by a discussion of the chief requirements of any model which explains the spatial and transient nature of these rare events. A simple model is then put forward incorporating the main features of the phenomena. The model is amenable to a mathematical analysis based on the field theory of charge drift. As many of the results as space restrictions permit are given. These are in agreement with the observations.

1. INTRODUCTION

Sightings of ball lightning are infrequent and most of us have never witnessed one of these rare events. When they have been observed, the phenomena are most eye-catching and notable, and the reported qualitative descriptions are in broad agreement with each other, although there are some significant differences. A good historical discussion is given by Wikipedia, and there is a review of the literature given in the New Scientist [1]. Early observations should be treated with caution because of the inherent inconsistencies and superstitions in the culture of those times. However, the events have several reported features in common.

A modern statistical account of ball lightning sightings in France taken over a 16-17 year period is available [2]. About 20 sightings per year are reported, each witnessed by typically about 5 people. Hence, using population data, there is circa a 0.02% chance of someone directly observing one of these events in their lifetime. This may be more accurate than the 1 in 20 of the USA's population who claim to have seen the phenomenon [3]. There are other inconsistencies: the glowing balls usually suddenly fade, but there are reports of explosive endings; and whereas there seems to be no danger to life by being close to a glow, deaths have been recorded in some early reports. The phenomena are usually observed as being spherical, some with tails, but there is an incidence of an annular glow being formed after one lightning strike [4].

The spherical glows almost invariably occur during or just after storms, particularly electrical storms with lightning strikes to ground. But there are some reports of events taking place when the skies are clear. These quasi-stable phenomena usually last for about 5 to 10 seconds, although there are some reports of them lasting for a few minutes. The diameters are generally a few tens of centimetres. Some are quite small (1 to 2 cm), and there are a few reports of diameters of a metre or slightly more. The physical sizes of the phenomena are reported as roughly constant during their lifetimes.

The spheres tend to be ground or surface hugging with heights of 1-2 metres. They move relatively slowly at a few metres per second, with some observers reporting a seeming independence of the movement of the surrounding air. As the glows are clearly electrical in their nature, their resultant motion must be a result of the ambient conditions which should include the local electric field and buoyancy effects because of possible heating as well as the macroscopic movement of the surrounding air. The spheres also seem to have a liking for metallic objects (railings etc.) which makes some sense because of the increased electric field near grounded objects. The luminosity of the objects seems to be of the order of 1000 lumens - essentially that of a 100W tungsten filament bulb. They are usually opaque and white or yellowish in colour, although there are a significant number of other colours, usually at the lower red end of the spectrum, with a few reports of blue or violet hues. Most observers say that there are distinct surfaces, but some tell of fuzziness and some of translucency. A large proportion of witnesses have been within a few metres of these glows. About 1 in 5 reports a crackling sound, and there is a significant minority of events that end with explosions or cracking noises (particularly those outdoors). There are even some reports of whistling. A third of observers smelled ozone or a sulphurous smell, although one wonders what proportion of people would recognize the smell of ozone. A large proportion of sightings have been made by indoor observers, and many have been very close to the glows which do not appear to be harmful to humans. There are a tiny number of reports of injuries, and the phenomena seem to veer away from people.

Perhaps the most astonishing property of ball lightning is that it can pass through doors and windows. There is a recent explanation of this based on the residual ions left by the stepped leaders of lightning strikes [5]. It has been shown by detailed calculations that ionic columns impinging on thin insulating surfaces (glass, wood, etc.) can give space-charge fields on the other sides of the surfaces sufficiently high to initiate electrical discharges that could cause ball lightning. There are recorded events of this happening aboard aircraft [6], with glows being formed at cockpit windows. The chances of this happening in modern times would seem to be lessened by metallic heating filaments in aircraft windscreens.

The diversity of the observations may be explained by there being more than one phenomenon with spherical glowing characteristics and, although this is unlikely, it must be borne in mind. Because they are so interesting, there have been several attempts to create the objects in the laboratory, but they have not as yet been satisfactorily reproduced and there are no quantitative measurements of these unusual physical happenings.

The exact physical mechanisms responsible for the events are not known; and a good scientific explanation is long overdue. A simple mathematical model of a glow discharge will be outlined here. It is necessary for any theory to be in agreement with the main observations, even though any model would be difficult to verify because of the elusive nature of the phenomena. The model is based on the established electrical theory of the movement of atmospheric ions, and the reasoning behind it is explained with references to published work. As much of the mathematical theory as the space restrictions allow will be given. There are some interesting conclusions which could be tested if ball-lightning is ever produced in controlled conditions. A listing of the generally accepted main features of these phenomena now follows.

2. ESSENTIAL FEATURES OF BALL LIGHTNING

(i) The phenomena occur during or immediately after electrical storms.
(ii) They consist of electrically active spherical regions emitting photons (~a thousand lumens) with some evidence of thermal activity.
(iii) Sizes are approximately constant with radii in the centimetre range.
(iv) Typical lifetimes are several seconds.
(v) No diminution of light intensity during lifetimes reported.
(vi) There are no external sources of energy.
(vii) They usually suddenly disappear without a change in aspect.

3. THE SIMPLE MODEL

Successful mathematical modelling may well start with the simplest possible scenario which, if shown to give accurate predictions, can be further modified. The benefit of simplicity is that it can lead to analytic solutions giving an understanding of the nature of the physical behaviour; whereas more sophistication can result in numerical problems whose solutions may not give such clear insights.

The fact that clouds of monopolar ions form spherical shapes in their self-fields is now established [7-9]. The ions form spheres more quickly than their outward dissipation by self-repulsion. Hence, it is proposed that monopolar spheres of charge could be formed by the extreme conditions during lightning activity. The polarity of these spheres is likely to be positive [5] and this is adopted here, although it is a simple matter of a sign change in the equations in order to accommodate negative spheres. On creation, the spheres are thought to be thrust into a uniform negative sea of ions. As they expand, the positive ions meet incoming negative ions in a surface layer. Here they interact so giving off photons. External electric fields, such as the earth's field which might explain the earth-clinging behaviour of the glows, are ignored in this simple formulation. It is interesting to note that positive ions last ~5 minutes after creation in air before returning to the level of their background density, but that negative ions spend the best part of an hour before doing so [10,11].

The analysis relates to an initially homogeneous positive sphere with total charge Q_0, density ρ_{+0}, volume D_0, radius R_0 with mobility μ_+ embedded in an infinite sea of background negative charge lying outside the sphere with initial density ρ_{-0} and mobility $\mu_- < 0$. As time t increases, the positive charges are driven outwards by the electric field $\mathbf{E}(r,t)$ where r is the radius from the centre of the sphere, so that they lie inside a volume D_+ with radius R_+. As they move, they will meet negative ions being pulled inwards by the field, so that interactions will take place releasing light and heat. The negative ions will be bounded internally by the domain D_- with radius R_-. The usual recombination law using a convective time derivative can be used; and the appearance of this glowing spherical shell will seem to have an approximately constant size because of the decrease in the number density of the positive ions as they drift outwards recombining with the negatives that they meet. Space restrictions make the provision of a diagram difficult, but the geometry of the situation can be envisaged without too much difficulty. This situation is amenable to mathematical analysis using vector field theory and some earlier published results.

4. SOME MATHEMATICAL PRELIMINARIES

The mathematical theory of how ions behave in a gaseous medium [7-9] is based on the results of vector field theory and the mobility hypothesis

$$\mathbf{v} = \mu \mathbf{E} \tag{1}$$

where $\mathbf{v}$ is the ionic velocity. The mobility coefficient μ takes the sign of the ions and can be μ_+ or μ_- in the above. For aerial ions, standard values are $\mu_+ = 1\cdot 4\times 10^{-4}\,\mathrm{m}^2/\mathrm{Vs}$, $\mu_- = -1\cdot 8\times 10^{-4}\,\mathrm{m}^2/\mathrm{Vs}$, but these will change with heating [12], so that the mobility of positive ions could be an order of magnitude greater than what is quoted here.

Without including recombination rates, the Lagrangian (or convective) continuity equation for a single ionic species can be written

$$\frac{\mathrm{D}\rho}{Dt} = -\frac{\mu}{\varepsilon}\rho^2 \tag{2}$$

where $\rho(t)$ is the charge density when moving with the ions and ε is the permittivity. This is the charge drift equation which has the solution

$$\frac{1}{\rho} - \frac{1}{\rho_0} = \frac{\mu}{\varepsilon}t \tag{3}$$

where ρ_0 is the original charge density. Using the above with the divergence theorem, it can be shown [7-9] that any volume $D(t)$ containing single-species ions of total charge Q, moving so that its surface is the envelope of the charges, obeys the linear expansion law

$$D(t) = D_0 + \frac{\mu Q}{\varepsilon}t \tag{4}$$

where D_0 is the initial volume. This can be modified to take into account a varying total charge inside $D(t)$ by replacing Qt by the integral of the total charge over time.

The above is a synopsis of the theory of charge drift which is a most useful approach to many of the problems of ionic flow. Here the sphere theorem can be cited which has been stated in the previous section. This of itself explains the spherical nature of ball lightning. An outline of how these methods can be further used to explain other properties of the phenomena is given as appropriate. Space restrictions mean that a detailed account cannot be given, particularly of the ionic behaviour in the spherical shell. However, a numerical critique of some of the results quoted here is given later.

5. SOME IMPORTANT RESULTS OF THE MODELLING

There are two analytically solvable ways of modelling the sea of negative charges surrounding the positive sphere. One regards the density of outer negative charges as constant with value ρ_{-0} say, the other uses the expansion formula (3) to give $\rho_-(t)$ at time t with some initial value ρ_{-0}. These are called Case A and Case B respectively. Case B is more realistic, as in reality the surrounding negative charge density will be spatially changing. The flux theorem and charge-drift methods can be used to give the following results after some basic calculus.

Case A. The total charges in the inner and outer spheres D_- and D_+ at time t can be shown to be

$$Q_+(t) = Q_0 \Big/ \left(1 + \mu_+ \rho_{+0} t/\varepsilon\right)^{\frac{\mu_+ - \mu_-}{\mu_+}} \tag{5}$$

$$Q(t) = Q_0 + Q_-(t) = Q_0 \exp\left(\frac{\mu_+ - \mu_-}{\varepsilon} \rho_{-0}\, t\right) \tag{6}$$

respectively; with the relevant volumes given by

$$D_-(t) = D_0 \left(1 + \mu_+ \rho_{+0} t/\varepsilon\right)^{\frac{\mu_-}{\mu_+}} \tag{7}$$

$$D_+(t) = D_0 \left(\begin{array}{l} 1 + \dfrac{\mu_+ \rho_{+0}}{(\mu_+ - \mu_-)\rho_{-0}} \times \\ \left(\exp\left(\dfrac{(\mu_+ - \mu_-)\rho_{-0}}{\varepsilon} t \right) - 1 \right) \end{array} \right) \tag{8}$$

from which the radii are easily found if needed.

Case B. $Q_+(t)$ and $D_-(t)$ are now as above, but

$$Q(t) = Q_0 + Q_-(t) = Q_0 \Big/ \left(1 + \mu_- \rho_{-0} t/\varepsilon\right)^{\frac{\mu_+ - \mu_-}{-\mu_-}} \tag{9}$$

$$D_+ = D_0 \left(1 - \frac{\rho_{+0}}{\rho_{-0}} \left(1 - \left(1 + \frac{\mu_- \rho_{-0}}{\varepsilon} t \right)^{\frac{\mu_+}{\mu_-}} \right) \right). \tag{10}$$

The limiting results with $t = 0$ and as $t \to \infty$ are as to be expected in both cases.

6. NUMERICAL VALUES, PHYSICAL COMPARISON AND CONCLUSIONS

As the above is an idealized model and ambient conditions will not be as perfect, predictions based on the theory as $t \to \infty$ are inappropriate. However, half-lives for the Qs can be obtained based on the analysis to see whether they give results that agree with the recorded observations. Values of ρ_{-0}s and also of the μs are required to effect this comparison. Normal negative charge densities are well known to be about $-5\times10^{-11}\text{C/m}^3$ [13], but in conditions near and during thunderstorms, number densities are known to be as much as two orders of magnitude higher [14,15]. The range adopted here takes $-5\times10^{-9}\text{C/m}^3$ as the other limit. Values of the μs are quoted earlier, but in the thermal conditions of the glow μ_+ values may be many times more than this [12], so that $10|\mu_-|$ has been taken as a sensible upper limit of the range.

It can be shown that the two cases give nearly the same half-lives; and taking $\mu_+ = |\mu_-|$ the half-lives have the approximate range 500s-5s using first the smaller and then the larger value of $|\rho_{-0}|$. Taking $\mu_+ = 10|\mu_-|$, this range is 100s-1s. These values are exactly what observers of these rare phenomena have reported, which gives considerable confidence to the theory. Unfortunately, space restrictions mean that details of the calculations cannot be quoted.

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