GAS DYNAMICAL MODEL OF LINEAR LIGHTNING DISCHARGE

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ABSTRACT

The hydrodynamic model of movement of the charged particles in a stream, expanding applicability of the initial model was studied. In present paper the analysis of various approximations 1D of model is carried out and phase trajectories of the electric field evolution induced by a stream and density of particles in a stream at various speeds of excitations are constructed.

1. INTRODUCTION

Atmospheric discharges are surprisingly interesting phenomena that occur in the surface layers of thunderclouds towards the ground, and rising from the tops of the clouds through the troposphere to the ionosphere - sprites, blue and gigantic jets, elves. Scales of discharges cover areas from 12-20 km up to 40-90 km [1-3]. At the heart of all these phenomena is the process of charge separation, the growth of the electric field and the subsequent breakdown of air. By itself, this phenomenon is random in time and space character, but preparation of the charge separation and subsequent growth of the electric field is a natural process, the cause of which is a set of different physical factors. The main processes leading to the emergence of linear lightning, suggested two mechanisms described in great detail, for example, in the excellent work [2]. In one of them conventional breakdown on the electrons warmed in electric field is offered. The second mechanism – breakdown on run-away electrons – is energetically more favorable as it implies much smaller values of a field for acceleration of electrons and creation of avalanches. In this paper, we propose a new approach to the explanation of the physical mechanism of discharge, based on consideration of the hydrodynamic flow of charged particles in a thundercloud, caused by natural causes of convection and capture of charged particles by the ascending streams in electric field of the Earth. The existence of hydrodynamic flow of charged particles in a weakly ionized gas can lead to instabilities in relation to the non-uniform charge distribution in the cloud, increase of the electric field and the occurrence of local regions in which the electric field exceeds the threshold value necessary for the breakdown at a given altitude.

2 . THEORY

We assume that we are dealing with a dense weakly ionized gas consisting of electrons, ions and uncharged particles. Based on estimates of the concentration of charged particles about 10^3 cm^{-3} and assuming approximately constant temperature in a layer of the ground atmosphere at the altitude of thunderclouds, it is possible to estimate the Debye radius $r_D = v_T/\omega_n \approx 4$ cm that exceeds distance between the charged particles and therefore at smaller distances there is no Coulomb shielding of the charged particles. Here $v^2_T = kT/m$ – the thermal velocity of electrons, and $\omega_n = 10^{12}$ s^{-1}– the characteristic frequency of fluctuations of the charged particles. As the main equations describing dynamics of streams of charged particles, we use Euler's equation, the equation of continuity and Poisson's equation for model of weakly ionized gas. In a thundercloud in the field of the Earth we will consider initial distribution of the charged areas negative in the lower part and positive in the top part, although the observed charge distribution is more complicated [2,4]. Temporal and spatial distribution of the electric field in the one-dimensional case will then be described by the system of equations:

$$-enE-\frac{\partial P}{\partial x}-mn\nu(\upsilon-\upsilon_0)=mn\frac{\partial \upsilon}{\partial t}+mn\nu\frac{\partial \upsilon}{\partial x},$$
$$\frac{\partial n}{\partial t}+\frac{\partial (n\upsilon)}{\partial x}=0, \qquad (1)$$
$$\varepsilon_0\frac{\partial E}{\partial x}=4\pi e(N_0-n),$$
$$J=en\upsilon$$

Here the following designations are entered: e – charge of a particle, m – its mass, υ – the particle speed , ν - the particle collision frequency with all scattering centers, n – concentration of particles, N_0 – concentration of particles of an opposite sign, υ_0 – the speed of neutral particles, ε_0 – dielectric permeability of media, P – gas pressure, in adiabatic approximation. We assume initially isothermal process at scales within the cloud and in times previous to the formation of the leader. Then pressure can be presented in the form of P = kTn. In this approach we don't consider mechanical viscosity, but electro dynamical viscosity is considered partially, by introduction of the member of collisions $mn\nu(\upsilon-\upsilon_0)$. Recharge mechanism of neutral particles and ions can be accounted for by adding the collision integral to the right side of the equation continuity. As a first approximation, we will not consider the process of recharge and will take it into account at further consideration. We will write down system (1) in a dimensionless form, using the following designations: $\xi = x/l$, $\tau = t/\tau_0$, $y = eEl/(kT)$, $\rho = nl^3$, $\rho_0 = N_0 l^3$, $u = \upsilon/(\nu l)$, $u_0 = \upsilon_0/(\nu l)$, where $l = e^\wedge 2/kT$ - characteristic distance at which energy of Coulomb interaction is equal to thermal energy of particles;

$$\tau_0=\frac{m\nu l^2}{kT}=\frac{\upsilon_D{}^2}{\upsilon_T{}^2\nu}\equiv\frac{l^2}{D}$$ - - time, during which non equilibrium concentration of particles is leveled due to diffusion (υ_D – the speed of diffusion, υ_T – the thermal speed of particles, D=kT / (mν) – diffusion coefficient). The resulting system can be written as:

$$-\rho y-\frac{\partial \rho}{\partial \xi}-\tau_0\nu\rho(u-u_0)=\rho\frac{\partial u}{\partial \tau}+\tau_0\nu\rho u\frac{\partial u}{\partial \xi}$$
$$\frac{\partial \rho}{\partial \tau}+\tau_0\nu\frac{\partial(\rho u)}{\partial \xi}=0 \qquad (2)$$
$$\frac{\partial y}{\partial \xi}=-\frac{4\pi}{\varepsilon_0}(\rho-\rho_0)$$
$$j=\rho u$$

For the dimensionless density of a stream of particles we use $j=Jl^2/(e\nu)$ designation is entered. Using the continuity equation, we exclude from this system of equations speed. After simple transformations we obtain the system of equations with two variables - the particle concentration and the magnitude of electric field in a gas flow:

$$\begin{cases}(y_0-y)\cdot\rho-\frac{\partial\rho}{\partial\xi}-c_1(\tau)-\frac{\varepsilon_0}{4\pi}\cdot\frac{\partial y}{\partial\tau}=\frac{1}{\tau_0\nu}\cdot\frac{\partial}{\partial\tau}[\\ c_1(\tau)+\frac{\varepsilon_0}{4\pi}\cdot\frac{\partial y}{\partial\tau}]+\frac{1}{\tau_0\nu}\cdot\frac{\partial}{\partial\xi}\left[\frac{1}{\rho}\cdot\left(c_1(\tau)+\frac{\varepsilon_0}{4\pi}\cdot\frac{\partial y}{\partial\tau}\right)^2\right]\\ \frac{\partial y}{\partial\xi}=-\frac{4\pi}{\varepsilon_0}\cdot(\rho-\rho_0)\end{cases}$$
(3)

Here we used $y_0 = \tau_0\nu u_0$ designation. The integration constant $c_1(\tau)$, received by us after integration of the equation of a continuity on coordinate is defined by values of density, speed of a stream and change of electric field on thundercloud border a ratio:

$$c_1(\tau)=\tau_0\nu\rho(\xi_0,\tau)u(\xi_0,\tau)-\frac{\varepsilon_0}{4\pi}\cdot\frac{\partial y(\xi_0,\tau)}{\partial\tau} \qquad (4)$$

This value is the continuous function of time becoming zero on the border, and nonzero outside of it due to the fact that change of electric field at a given point in space during a period is compensated by density of a stream of the charged particles through thundercloud border. Tangential components of a rotor of a vector of magnetic field at the border do not undergo the jump, and will not affect the one-dimensional motion of the current density. We will enter the new ζ variable associated with old variables by coordinate of ξ and time of τ by relation of $\zeta = \xi - V\tau$. Considering ratios

$$\frac{\partial}{\partial\xi}=\frac{d}{d\zeta},\quad \frac{\partial}{\partial\tau}=-V\frac{d}{d\zeta}$$

we will write down system (3) in its final form:

$$\begin{cases} \frac{d\rho(\zeta)}{d\zeta}\cdot\left[\rho^2(\zeta)-\frac{(c_1(\zeta)-V\rho_0)^2}{\tau_0\nu}\right]+\rho^2(\zeta)\cdot \\ \left[c_1(\zeta)+V[\rho(\zeta)-\rho_0]+\rho(\zeta)[y(\zeta)-y_0]-\frac{V}{\tau_0\nu}\cdot\frac{dc_1(\zeta)}{d\zeta}\right]=0 \\ \frac{dy(\zeta)}{d\zeta}=-\frac{4\pi}{\varepsilon_0}\cdot[\rho(\zeta)-\rho_0] \end{cases}$$

(5)

The system (5) describes distribution of a stream density and electric field evolution in one-dimensional approximation and at the set function $c_1(\zeta)$, defined by the conditions on the thunderstorm border. It allows to calculate distribution of electric field and density of a gas stream with a change in a parameter V – speed of excitation of a field concerning the system of coordinates moving with a speed of a gas stream. In the approximation of $\rho_0 = 0$ the system can be integrated to obtain the equation

$$\frac{dz}{d\zeta}=-\frac{1}{4}(z^2-2b\zeta-2c-a^2)$$
$$\pm\frac{1}{4}\sqrt{(z^2-2b\zeta-2c-a^2)^2-16d^2}$$

$$z=y+a,\ a=V-y_0,\ b=\frac{4\pi}{\varepsilon_0}c_1,\ c=b\cdot\frac{V}{\tau_0\nu},\ d^2=\frac{1}{\tau_0\nu}\cdot b^2$$

Considering that d2 ≤ z2 we will receive equation of Riccati

$$\frac{dz}{d\zeta}=-\frac{1}{2}(z^2-2b\zeta-2c-a^2) \qquad (6)$$

which has the decision

$$z(t)=-\frac{1}{A}\cdot\frac{u_t'}{u},\ \text{где}$$

$$u(t)=\sqrt{t}\cdot\left[C_2\cdot J_{\frac{1}{2q}}\left(\frac{1}{q}\sqrt{ABt^q}\right)+C_3Y_{\frac{1}{2q}}\left(\frac{1}{q}\sqrt{ABt^q}\right)\right] \qquad (7)$$

$J_m(x), Y_m(x)$ - Bessel functions of the first and second kind. $q=\frac{n+2}{2}$, in our case n = 1. The dependence of an electric field magnitude as functions of coordinate is shown in Figure 1.

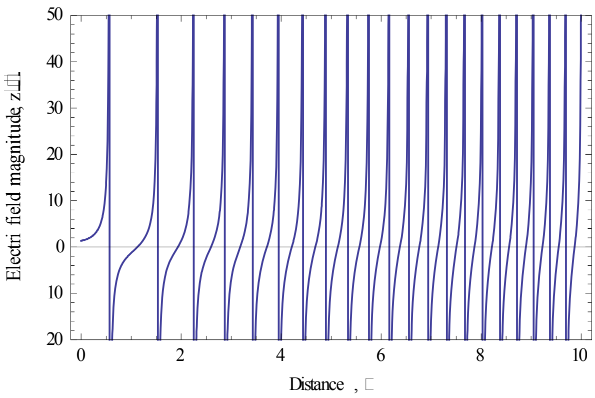

Fig. 1.The dependence of z (ζ) at values of the constants C1 = 1.2, C2 = 2.3, found from boundary conditions. The constant b = 10, a variable ζ runs values from 0 to 10.

In the resulting approximation, we have a significant increase in the electric field at the stream movement, and there are gaps in the field with poles, condensing with the increase of the stream coordinates. We will consider system (5) without any approximations. As the first step we express dimensionless concentration of ρ(ζ) from the second equation and substitute in the first equation. Then we receive the equation

$$y_{\zeta\zeta}(A_1y_\zeta^2+B_1y_\zeta+C_1)=(A_2+y)\,y_\zeta^3+(B_2+y)y_\zeta^2+(C_2+y)y_\zeta+D \qquad (8)$$

If we enter the new variable $w(y)=y_\zeta$, including y as an independent variable then the equation (8) can be write in a form:

$$w_yw(A_1w^2+B_1w+C_1)=(A_2+y)\,w^3+(B_2+y)w^2+(C_2+y)w+D$$

If we rewrite it in a form

$$dy[(A_2+y)\,w^3+(B_2+y)w^2+(C_2+y)w+D]=dw\,[w(A_1w^2+B_1w+C_1)], \qquad (9)$$

that it can be integrated after finding of an integrating multiplier. We will not write the solution of this equation because of its complexity, and present only the graphs obtained by us in solving this equation in y(ζ). Figures 2 - 5 show the dependences of the given concentration, magnitude of electric field from the coordinates, entry conditions and various values of excitation velocity in a stream of ionized gas.

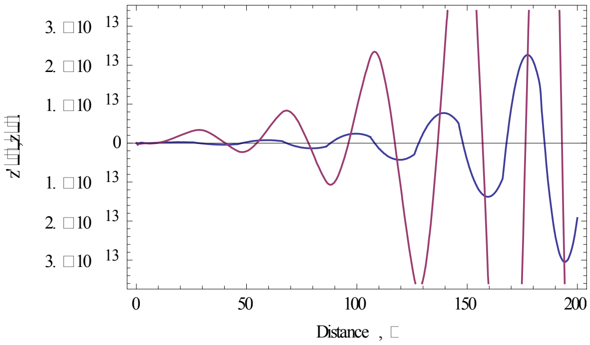

Fig. 2. Evolution of electric field magnitude in a stream (red line) and the derivative of the field in a stream (the blue line) with the speed of excitation in a stream of V = 2265.32, constructed in the range of variation of the specified coordinate of ξ = [0 – 200].

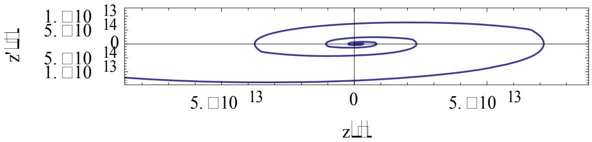

Fig. 3. Phase trajectory of the electric field magnitude y corresponding to figure 2 data y' (y) at values of the specified coordinate of ξ = [0 – 200].

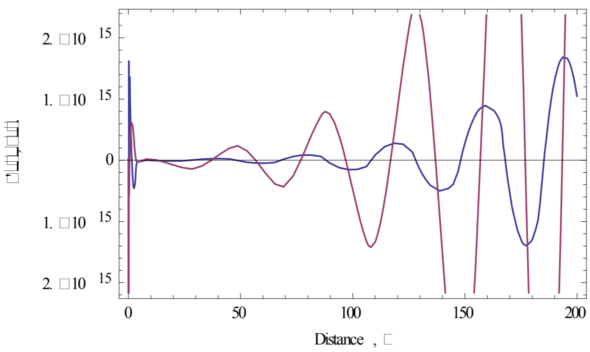

Fig. 4. Evolution of a stream density (the red line) and the derivative of a stream density (the blue line) at value of speed of excitement in a stream of V = 2265.32 and areas of change of the ξ variable = [0 – 1000].

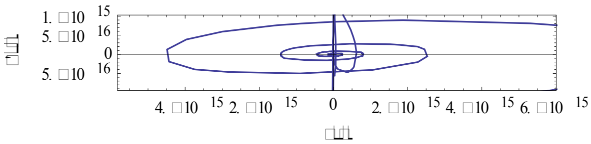

Fig. 5. Phase trajectory of a stream density of ρ'(ρ) corresponding to figure 4 data at values of the specified coordinate of ξ = [0 – 1000].

Amplitude of electric field increases by 5 – 7 orders, remaining limited as the speed of stream increases. As the results of our research it is possible to make the conclusion that the used gas dynamic model of formation of the linear atmospheric discharge can even describe the emergence of the leader connected with transformation of kinetic energy of a stream in energy of the discharge at the expense of creation of high electric fields in a stream in the considered one-dimensional case.

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